Have You Heard of JIU ZHANG SUANSHU?

Jiu Zhang Suanshu stands out in the world history of mathematics. It is the earliest most outstanding book on arithmetic that was built on two mathematical notations still indispensable today, one to express a number and the other to denote a common fraction.

by Lam Lay Yong

Our arithmetic is built on the Hindu-Arabic numeral system. We know how useful this arithmetic is to us. It not only provides a foundation to mathematics, it is commonly known to most people throughout the world as its knowledge is necessary in our everyday living. Try to replace the Hindu-Arabic numeral system with another numeral system of a different concept and you will find that our arithmetic and its applications will collapse like a pack of cards.

Since the numeral system is so important, let us examine it in detail. The system uses a place value notation with ten as base so that anyone who uses the numeral system will only have to remember the nine signs for numbers one to nine. Depending on the number, the nine signs or digits are picked and arranged in a horizontal line from left to right in descending order of rank. For instance, the number fifty six thousand nine hundred and thirty four is notated as: 56934, and the ranks of the digits from left to right are: ten thousands, thousands, hundreds, tens and units. In the very early Hindu-Arabic numerals, if a number had no digit of a certain rank, the space of that rank was left vacant so that fifty six thousand nine hundred and four would look like this: 569 4. It was later that this empty space was filled with the zero symbol as we know it today.

From 1200 to 1600, the peoples of Europe discarded their own numeral systems or, like the Romans, displaced them to secondary importance, and laboriously started to learn the Hindu-Arabic numeral system. They had before this found even simple multiplication and division extremely difficult to perform, and knew that the new numeral system would open for them an exciting world of computations leading to the new arithmetic, which would be very useful in many areas and especially in commerce. They probably had the same feeling as we have now about computers opening a new vista of high technology for us.

The numeral system produced a method of division which resulted in a notation for the common fraction; for example, five sevenths was expressed as: $\frac{5}{7}$, without the horizontal line which we use today. Based on the knowledge of the numeral system and the notation to express a fraction, a new world of computations began to unfold. Literature on arithmetic grew phenomenally and some of the common topics and methods included fractions, exchange of goods, partnership and sharing, proportion, Rule of Three, areas, volumes, the extraction of square and cube roots, and Rule of False Position.

In their attempts to compute, the ancient Chinese used a bundle of bamboo sticks or rods. Through this usage they invented a numeral system, which had the same concept as the Hindu-Arabic numeral system. The nine signs that represented the first nine numbers were:

𝍠 𝍡 𝍢 𝍣 𝍤

𝍥 𝍦 𝍧 𝍨

Like the Hindu-Arabic numerals, the digits were arranged in a horizontal line from left to right in decreasing order of rank. As the digits were formed from rods, the ancient Chinese had an ingenious device in displaying digits which occupied alternate positions. They turned a vertical rod horizontal and a horizontal rod vertical. For example, fifty six thousand nine hundred and thirty four would look like this: 𝍤𝍮𝍨𝍫𝍣. If a number had no digit of a certain rank, the space representing that rank was left vacant, so that fifty six thousand nine hundred and four would appear like this: 𝍤𝍮𝍨 𝍣. This was a very natural process for a system which handled with rods. The Chinese also invented the division method which left a remainder, and used the remainder in rod numerals to denote the complex concept of a fraction. For example, five sevenths was expressed as: 𝍤/𝍦.

Over two thousand years ago the Chinese were aware of two very useful notations - the numeral system that used a place value notation with ten as base and the notation to express a common fraction. Through the use of these two notations they were able to compute and develop numerous mathematical methods. Around the first century, such problems and methods were compiled into a book called *Jiu Zhang Suanshu* 九章算術 (Nine Chapters on the Mathematical Art).

Jiu Zhang Suanshu has nine chapters and two hundred and forty six problems. It begins with the topics on fractions and although the methods involve the manipulation of rods, they are surprisingly very similar to the methods that we use. This is followed by problems on areas of different shapes such as an isosceles triangle, a trapezium, a circle and an annulus. *Jiu Zhang Suanshu* has probably the earliest general description of the Rule of Three. This rule is first applied to problems involving exchanges of foodstuff and then to other cases. Problems on partnership and sharing dominate Chapter Three, while Chapter Four is concerned mainly with the methods of finding the square root and the cube root of a number.

The next chapter involves the calculation of volumes of solids such as a circular cone, the frustum of a cone, different types of wedges and a prism whose cross-section is a trapezium. Chapter Six is concerned with the application of proportion and inverse proportion, and gives a wealth of information on the socioeconomic aspects of life in ancient China. The first problem is stated below:

Now there is a fair [way of] transporting millet: County A has 10,000 households and [requires] 8 days' journey to reach the destination; County B has 9,500 households and [requires] 10 days' journey; County C has 12,350 households and [requires] 13 days' journey; County D has 12,200 households and [requires] 20 days' journey. The four counties transport a total of 250,000 *hu* 斛 of millet as tax and use 10,000 carts. It is desired that the contributions be based on the distances and the number of households. Find the amount of millet and the number of carts from each [county].
Answer says: County A 83,100 *hu* of millet, 3,324 carts. County B 63,175 *hu* of millet, 2,527 carts. County C 63,175 *hu* of millet, 2,527 carts. County D 40,550 *hu* of millet, 1,622 carts.

Besides problems on proportional parts, the chapter also has problems involving relative distance and speed. The Rule of False Position was one of the methods devised by ancient man to solve a problem at a time when his mind was unable to formulate or to think abstractly in terms of mathematical notations. The Chinese called the method *ying bu zu* 盈不足 and Chapter Seven is devoted to this topic. Chapter Eight involves the solutions of simultaneous linear equations up to six unknowns. The data are set in columns like our matrix notation; the subtraction of two columns gives rise to the concept and definition of a negative number. The final chapter deals with problems on the right-angled triangle.

Jiu Zhang Suanshu is the most important of all the very early Chinese mathematical texts that have survived. It provided a firm foundation and had a strong influence on the subsequent development of mathematics in China which reached its zenith in the thirteenth century.

Jiu Zhang Suanshu also stands out in the world history of mathematics. It is the earliest most outstanding book on arithmetic that was built on two mathematical notations still indispensable today, one to express a number and the other to denote a fraction.

When you read new books on the history of mathematics, you will notice a significant change from the old ones: *Jiu Zhang Suanshu* is gradually being given its rightful place of importance. This trend will continue into the next century as knowledge of the book increases. M²

References

Lam Lay Yong, "Jiu Zhang Suanshu (Nine Chapters on the Mathematical Art): An Overview" Archive for History of Exact Sciences **47** (1994), 1-51.

Qian Baocong 錢寶琮 (ed.) 1963. Suanjing Shi Shu 算經十書 (Ten mathematical classics). Beijing: Zhonghua shuju.

Lam Lay Yong is Professor of Mathematics at National University of Singapore. Her research area is history of Chinese mathematics.